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The author of the following article, Mr. Yoshio Mikami, has already acquired an enviable reputation as a student in the history of mathematics of Japan. Last year he published in the *Abhandlungen zur Geschichte der Mathematischen Wissenschaften* a work entitled "Mathematical Papers from the East." In this work he has included upwards of fifty articles showing the present interest of mathematicians in Japan, not merely in the history of mathematics, but in analysis, theory of numbers, modern geometry, theory of functions, and other topics of this nature. He is a student and friend of the learned historian of mathematics, Mr. Endo. He has not connected himself with any of the faculties of the Japanese universities, preferring to live the life of an independant savant. He has contributed extensively to the *Bibliotheca Mathematica* and the *Nieuw Archief voor Wiskunde* and ranks with Professor Hayashi as one of the best known of the younger historians of mathematics in the East. Having myself become much interested in the history of Oriental mathematics, and having made a large collection of early printed works and manuscripts on this subject, it was a pleasure to me to have Doctor Paul Carus suggest that I collaborate with Mr. Mikami in a history of Japanese mathematics. This I have done, and it is expected that the work will appear during the coming year. Mr. Mikami has made some valuable translations that set forth the nature of the native mathematics in his country. He is an indefatigable worker, and the West is already much in his debt and will be much more so in the next few years.

DAVID EUGENE SMITH.

THE TEACHING OF MATHEMATICS IN JAPAN.

By YOSHIO MIKAMI, Okara in Kazusa, Japan.

For the last two or three centuries there flourished a peculiar style of mathematics in Japan, which we distinguish from the European mathematics by the name *Wasan* or Japanese mathematics. It was first learned from China, yet further developments were effected quite independent of the Chinese.* But Japan saw a political change in 1868, which is well known

* As to the history of Japanese mathematics, a work will shortly appear under the joint authorship of Dr. David Eugene Smith and Yoshio Mikami.

as the *Ishin* or Restoration. The country was opened to commerce with Western powers and the Occidental civilization then poured in. Old schools were closed and many new ones were established, where mathematics were taught in the Western style, Japanese mathematics of former days being neglected. Those who had been schooled in the old style were obliged to learn the new science, and the state of things has changed in a short time to assume an appearance in accord with the rest of the civilized world. At present only Occidental mathematics is taught in the schools. The only exception being the teaching of the *soroban* arithmetic in common schools. The *soroban* is an abacus that was brought from China some centuries ago and that has been used universally during the ages of the old Japanese mathematicians.

The schools established in Japan since the Restoration have often gone through reorganizations, but this is not the place to give details about these changes. Suffice it to say, the main system of education has always remained untouched in spite of the frequent changes, being composed of the common school, the middle school, and the university. At present there is the higher school between the last two.

The common school provides ordinary and higher courses, the former extending through six years and the latter through two or three additional years. When boys and girls reach the age of six they are required to attend the ordinary common school and to remain there for six years. The arithmetic taught in this school consists of the following subjects:

First year: Numeration of numbers up to 100. Mental addition, subtraction, rudimentary multiplication and division.

Second year: Mental calculations with numbers up to 1000, including the four operations.

Third year: Numeration of numbers up to 10,000. The four operations in writing.

Fourth year: Four operations on numbers up to a hundred million. Calculations with various measures. Decimal fractions.

Fifth year: Four operations on whole numbers and decimals. Areas and volumes. Arithmetic with various measures; the metric system, foreign measures.

Sixth year: Four operations with fractions. Problems in ratio and rates. The *soroban* arithmetic may be taught in this year, but it may also be dispensed with. In the majority of schools, however, it is actually being taught.

It must be admitted that the Japanese system of numeration is much simpler than in the newer form expressed in the European languages, and this is a reason why the young Japanese are taught more arithmetic than is the case in Europe and America. The class-books of common schools are all prepared by the State under the direction of the Department of Education. The chief editor for arithmetical treatises is S. Iijima.

The compulsory education ends with the ordinary common schools, and the graduates of these schools are admitted to middle schools, though not infrequently an examination is required on account of the large number of applicants. The higher common school is intended for those who have to complete their education here. Its course regularly consists of two years, but it may extend to three years. The arithmetic in this school is roughly as follows:

First year: Greatest common measure, least common multiple, calculations with fractions in a more general sense than in the ordinary course, problems of application, problems in rates, simple and inverse proportions, and percentage partition.

Second year: Proportion, including compound proportion; drill in review of previous work. Geometrical drawing is also taught.

In the *third year*, when given, some further lessons in arithmetic are provided.

The middle schools extend through five years, the graduates of ordinary common schools being admitted. The lessons in mathematics are distributed somewhat as in the following table:

	1st Year	2nd Year	3rd Year	4th Year	5th Year
Arithmetic	3 3 3	3 3			
Algebra		3	2 2 2	2 2 2	2
Geometry			3 3 3	3 3 3	2 2 2
Trigonometry					2 2

The figures indicate the number of hours per week and the dotted lines indicate the divisions of terms, one school year consisting of three terms.

It is a matter of course that the distribution of lessons varies according to the different schools. The above is only a specimen.

In arithmetic, the following subjects are taught: Calculation of whole numbers, properties of the same, calculation of fractions, decimals, measures, money, etc. Simple, inverse, and compound proportions, problems in rates, percentage, profit and loss, interest.

The algebra course is somewhat similar to that given in the briefer of

Charles Smith's two treatises; including the four operations, indices, linear operations in one unknown, linear simultaneous equations, fractions, quadratic equations, irrational equations, extraction of square and cube roots, proportion, arithmetic and geometric progressions, interest, etc.

The geometry course is modeled after the treatise published by the Association for the Improvement of Geometrical Teaching in England. Proportion is treated according to the algebraic method. Solid geometry follows closely the outline of geometry in Wilson's treatise.

The trigonometry course follows closely the contents of John Casey's little work.

The text-books of the middle school are not prepared by the State as in the case of the common school, but are left to private authors. Nevertheless they are examined by the Department of Education, before they are authorized. There is a considerable number of these text-books now in use in Japan. Among those the most popuular are D. Kikuchi's Geometry, R. Fujisawa's Arithmetic, the same author's Algebra, etc. We are greatly indebted to the authors of these able treatises for the success of mathematics teaching in our country.

In the Japanese middle school all the boys are taught in the same manner, even though they may differ widely in their aims. Those who have to complete their education in the school and those who are preparing for higher grades of education are all instructed in the some school and all in the srme manner. Even the literary and scientific courses are not distinguished. Consequently the number of subjects taught becomes exceedingly large. And, moreover, the interrelations between these numerous subjects receive little attention. Even the different branches of mathematics are taught quite independently and irrespective of other branches. We hope however that such a defect as this will be remedied in the future, as a new program for the work of the schools is being prepared by the Department of Education, which will be made public very soon.

Before the graduates of the middle school may enter the university they must go through another school, whose course lasts three years. This school and other higher professional schools are all open to the graduates of middle schools, but as there is an insufficient number of these schools the candidates are obliged to pass competitive examinations, which proves a great barrier for young men. On this account they are naturally induced to pay most attention to the results of their examinations and the standing of these schools is determined solely by this criterion. It results therefore that the lessons are practically regulated with the sole aim of preparing for these examinations, which have thus acquired such an importance that they directly influence the teaching of the middle schools. It is consequently necessary to say something about these examinotions. As specimens we quote the problems given last summer in the entrance examination for the higher schools.

Algebra. 1. If m arithmetic means be inserted between two numbers a and b, it is required to find the rth term counting from a on.

2. A certain man leased land for 144 yen, and keeping 18 units of it for his own cultivation, he sublet the rest to a third man at a gain of 0.20 yen per unit of land. Then the amount he receives just paid his own rent. Required the number of units of land leased by him.

3. Given the relations $\frac{bz+cy}{b-c}=\frac{cx+az}{c-a}=\frac{cy+bx}{a-b}$, prove that $(a+b+c)(x+y+z)=ax+by+cz$.

4. Determine the values of p, q, r such that the coefficients of x^5, x^3, and x shall vanish ln the expansion of $(x^3-px^2+qx-r)(px^3+x^2+5x+7)$.

Geometry. 1. If from a point P outside a circle whose center is C two tangents PA and PB are drawn to the circle, and if any chord MN is drawn through the middle point of the chord AB, it is required to prove that the four points P, C, M, N are concyclic and that the line joining the point P and the center C bisects the angle MPN.

2. Prove that the area of a quadrilateral is constant, if the lengths of its two diagonals and the angle included by them are constant.

3. There is a solid angle with three faces, whose diedral angles are each right angles. If the solid angle be cut by a plane, the orthocenter of the triangle formed by the intersections of the cutting plane and the faces of the solid angle is the same point as the foot of the normal dropped from the vertex of the solid angle to the plane.

Trigonometry. This examination included also questlons in trigonometry which are not here given.

Since the examinations are held for the purpose of making selections from among the graduates of middle schools, they ought to be in accordance with the subjects taught in these schools. But we are told that sometimes little attention is paid to this. As a rule, emphasis is laid on arithmetic, though nothing in this line has appeared in the above problems for entrance to higher schools. As the subject is taught only in the first years of the middle school, those who go to the examination rarely remember correctly what they have learned so long before. Therefore there are schools where arithmetic is repeated in the highest class or the fifth year, which proves a useless waste of time and labor for the students. For what need is there of using the purely arithmetical knowledge for those who are able to treat the same more simply by the aid of algebra?

Some time ago R. Fujisawa powerfully and ably maintained that arithmetic is the most difficult subject to teach among the various branches of mathematics, and consequently that most attention should be paid to it. This reasonable claim was unfortunately misunderstood as if meant to lay more stress on the subject of arithmetic than on algebra. The result was unfortunate, and we hope the abuse will soon be removed.

In the common school boys and girls are taught alike, but boys only are admitted to the middle school. Corresponding to the latter, however, there are girls' high schools, whose courses are not unique, consisting of three, four, or five years according to circumstances. The girls mostly finish their education in these schools, which are open to the graduates of the ordinary common school. The branches of mathematics taught in the girls' high school are, arithmetic, algebra, and geometry,—all the lessons being simpler than in the middle school. Sometimes algebra or geomety is withdrawn, and solid geometry is never given.

In a certain one of the girls' high schools mathematics is taught two hours per week to all the classes, as follows:

First year: Whole numbers and decimals, various measures, fractions.

Second year: Whole numbers, fractions and decimals, ratio and proportion, percentage.

Third year: Ratio and proportion, percentage, extraction of square root.

Fourth year: Elementary algebra.

Fifth year: Elementary geometry.

The normal school is a school of almost the same grade as the middle school and the girls' high school, and it is here that the teachers of common schools are trained. Most of the normal schools are so constituted that both boys and girls are admitted, although they are taught in different classes; but there are some schools established for boys or girls only. A program for the lessons in mathematics of the normal school was issued recently by the Department of Education. In it we find the interrelation of the various branches of the science emphasized, and the whole has been made more practical, certainly no small advancement in the teaching of mathematics. We hope that the program for the middle school will be arranged with the same aim.

The Higher Normal Schools, which are located in Tokyo and Hiroshima, are schools where the male teachers of the middle class schools are taught. The graduates of the normal school or of the middle school are admitted, and the preliminary course consists of one year, and the main course of three years. The preliminary course is the same for all students of the school, and the mathematics consists of arithmetic, algebra, and geometry, each two hours per week. These lessons are reviews of what has been given in the middle school or in the normal school.

In the main course there is a department for mathematics, physics, and chemistry. The teachers of mathematics are trained here. The department is subdivided into two different classes,—in the first of which mathematics and physics are mainly taught, while in the other physics and chemistry are emphasized.

The subjects of mathematics taught in the first class are as follows:

First year: Algebra, two hours, including series, continued fractions and determinants. These are treated a little more concisely than in C. Smith's *Treatise on Algebra.*

Geometry, two hours, comprising the advanced part of elementary geometry, together with modern geometry. Also trigonometry, two hours.

Second year: Theory of equations, two hours; analytic geometry, four hours, together with seminary exercises.

Third year: The calculus, six hours; solid analytic geometry and dynamics.

In the second class mathematics is taught in a more practical way, its lessons ending in the second year.

F. Sembon and T. Hayashi are the chief professors of the Tokyo Higher Normal School. T. Takehashi teaches in the Hiroshima School.

There are two other schools of the same nature for young women, one in Tokyo, and the other at Nara. In the Tokyo school the students are taught in three separate departments, — letters, science, and arts, each lasting for four years. Mathematics is taught in the science department, and the subjects are arithmetic, algebra, geometry with geometrical drawing, trigonometry, and the teaching of mathematics. These lessons are distributed among the four years, four hours per week in the first year, and three hours in other years.

In the Nara school the science department is subdivided into two courses, in one of which mathematics, physics, and chemistry are taught for the main subjects, and in the other mainly history and mathematics. The same subdivision is to be adopted in the Tokyo school in the coming academic year. The professor of mathematics in the Tokyo Female Higher Normal School is L. Mori.

Besides the main courses in these normal schools sometimes special courses are opened, where the training will end in two or three years.

Further, there is at Sendai a Temporary School for Training Mathematical Teachers. The course extends through two years. K. Hakii is the professor.

Teachers of middle class schools are also granted certificates by special examinations held by the Department of Education. Graduates of middle class schools only are permitted to apply for them. They are divided into two parts, preliminary and final. The former is held by the local governments and the latter by the Department itself. Only those who have been successful in the preliminary examination can take the latter. In mathematics the knowledge of arithmetic, algebra, and geometry is tested in the preliminary examination, and the first part of the final examination is also devoted to the same subjects. Only those who pass the earlier examinations are admitted successively to the examinations in trigonometry, analytic geometry, and the calculus. A terchers' certificate is granted to those who have succeeded in arithmetic, algebra, and geometry. These examinations are the same for men and women, except that the candidates for teachers of female schools are permitted to answer a smaller number of the questions. A few only go up to the calculus.

Those who take these examinations come from various quarters, but for mathematics the Physics School in Tokyo is the main source of supply. This is a private school. The classes extend through three years or six semesters. In the first two years all the students are taught together, while in the last year, the courses are subdivided, the first in mathematics and the other in physical sciences. Mathematics is taught there in the first two years from arithmetic and geometry up to the calculus, and in the third year some more advanced topics of the same subjects are taught. To speak more definitely, these subjects are as follows:

First year: Arithmetic, including theory of prime numbers, etc.;

elementary algebra, including quadratic trinomials, determinants, etc.; elementary geometry, elementary trigonometry.

Second year: Elements of analytic geometry, plane and solid; spherical trigonometry; differential and integral calculus, including algebraic analysis and differential equations.

Third year: Arithmetic and algebra, or theory of numbers and theory of equations, etc.; modern geometry, analytical trigonometry, differential and integral calculus.

Besides these, physics, chemistry, astronomy, and surveying are taught.

The teachers in mathematics are S. Hayashi, Kanazawa, Kariya, Kuniyeda, Mimori, Noguchi, Ogura, Sakuma, Sakurai, Sawayama, Sembon, Terao, and Yasuda.

In the teachers' college of the Waseda University, Tokyo, which is a private school, mathematics is taught in the mathematics department and also in the physics and chemisty departments. In the first of these departments the subjects taught are as follows: Arithmetic, algebra, geometry, trigonometry, which are distributed throughout the whole of three years. Plane analytic geometry in the first year, and solid geometry in the second year. Differential calculus in the second year, and integral calculus including differential equations in the third year.

In the second of the departments, arithmetic, algebra, geometry, trigonometry, and analytic geometry are taught in the first year; solid analytic geometry, and the differential calculus in the second year; and integral calculus in the third year.

The higher schools, eight in number, are preparatory to the Imperial Universities, and are subdivided into three departments. Of these the second department is preparatory to the colleges of science, engineering and agriculture. There mathematics is taught as follows:

First year: Algebra, three hours; and plane trigonometry, two hours.

Second year: Conic sections, three hours; and theory of equations, one hour.

Third year: Calculus, four hours; and dynamics, two hours.

In these subjects the following works are used as text-books or rather as books of reference: C. Smith's *Treatise on Algebra,* subsequent to the chapter on combinations; Todhunter's *Plane Trigonometry;* Puckle's *Conic Sections;* Aldis's *Solid Geometry* (about one-fourth of the book); Burnside and Panton's *Theory of Equations* (the first half); Williamson's treatises on differential and integral calculus, or those of Todhunter; McGregor's *Dynamics.*

Mathematics is not taught to the agricultural students in the third year.

The higher schools are to be reorganized in the near future by the Department of Education although the plan proposed is contrary to pub-

lic opinion. According to the plan the schools will be made institutions where higher general education is given independently of the preparatory work for entrance to the universities, although the graduates are to be received in the latter as before. In that case the courses of instruction will be largely altered, the subject of mathematics being not excepted. The consequence will certainly be a great change in the teaching of the middle school.

The preparatory course of the private Keio University, Tokyo, corresponds to the first department of the higher school, but here mathematics is largely taught with the aim of training the students in the habit of logical thought. This is not generally done elsewhere. The course extends through two years, in the first of which analytic geometry is taught, and in the second the calculus. The chief teacher is S. Fukugawa.

There are two Imperial Universities in Japan, one in Tokyo, and the other in Kyoto. In the mathematics department of the College of Science of Tokyo the following subjects are taught:

First year: Calculus, five hours; analytic geometry, two hours; miscellany in elementary mathematics, two hours; astronomy and method of least squares, three hours; theoretical physics, mathematical seminary, three hours.

Second year: Theory of functions and elliptic functions, three hours; algebraic curves, three hours; differential equations, two hours; theory of numbers and higher algebra, four hours; dynamics, three hours; mathematical seminary and physical experiments.

Third year: Theory of functions and elliptic functions, three hours; higher geometry, two hours; algebra, three hours; differential equations, two hours; miscellany in higher analysis, two hours; dynamics, calculus of variation, three hours; mathematical lectures (optional).

In the departments of astronomy, theoretical physics and experimental physics, mathematics is also taught. In the first year of these departments the following subjects are taught: Calculus, geometry, mathematical exercises, method of least squares. In the second year mechanics, differential equations, etc. In the third year of the first two departments lessons in the theory of functions are given.

In the department of chemistry mathematics is taught for three hours in the first year.

The professors of mathematics are R. Fujisawa, T. Takagi, E. Sakai, and T. Yoshiye, and assistant professor S. Nakagawa.

In some classes of the Engineering College mathematics is being taught somewhat after the practical fashion of John Perry, formerly of the College.

In the Science and Engineering College of Kyoto there is a department of mathematics, wherein are taught the following subjects: Plane and solid analytic goemetry, higher geometry, calculus including differential

equations, theory of functions, higher algebra, theory of numbers, mechanics, theoretical physics, astronomy, physical experiments, and special lectures.

These subjects are taken by the students at convenient times. They are not prescribed for the given years as in the Tokyo University, certainly presenting some advantages over the latter.

The mathematical courses in the departments of engineering are: Integral calculus, three hours for four months; differential equations, two hours for six months; dynamics, two hours for a year; theory of errors, two hours for four months.

The instructors are professors K. Miwa, J. Kawai, assistant professors S. Yoshikawa and T. Wada, and lecturer T. Nishiuchi.

The College of Science of the North-Eastern Imperial University will be opened at Sendai in 1911, with T. Hayashi and M. Fujiwara for professors of mathematics.

In the privately founded universities there is established no colleges of science except in Waseda, the teachers' course of which we have already mentioned. The Tokyo University for Women contains a department for science, which may be mentioned as corresponding to the same in the Normal School for Women, though there will be found some differences in the subjects taught.

The mathematical teaching of some professional schools other than the universities, may also be of some interest.

Technological schools are established in Tokyo, Nagoya, Osaka, Kyoto, and Kumamoto, where the graduates of middle schools are admitted. The mathematics taught in these schools consists of: Algebra, supplementing the knowledge learned in the middle school and including combinations and permutations, series, logarithms, and differential coefficients; solid geometry and important curves; conic sections and solid analytic geometry; the rudiments of the calculus, sufficient to give familiarity with the symbols.

These subjects are taught in some departments only, and in others they are omitted wholly or partly. The head master of mathematics in the Tokyo school is M. Mimori.

The Higher Commercial School of Tokyo provides a one-year preparatory course, and a three-year main course. In the former, mathematics is taught three hours per week. The subjects are: Measures, proportion and interest, problems in equations, series, logarithms and their application, probability, etc. Commercial arithmetic is taught in the first two years of the main course, the professor of mathematics being G. Sawata. There are several other schools of the kind in the country.

Mathematics is not taught in the Military Officers' School but is given in the Military School for Artillerymen and Engineers, and also in the Military College. The first of these schools is the institution where the officers of the army are trained, the other two being intended for the completion of

the officers' education; the Military College produces officers of the staff. The mathematics taught in these schools concludes with the rudiments of conic sections and the calculus.

In the naval schools there are also courses in mathematics. In the Naval Engineering School, at Yokosuka, where three and one-third years are required for graduation, the following subjects are prescribed: Algebra, geometry, trigonometry, analytic geometry, calculus, elementary dynamics, and applied mechanics. A similar plan for the mathematics courses is followed in the Naval School at Yetejima, where all the Japanese naval officers are trained. In the Naval College at Tokyo some courses in mathematics are also given, K. Ashino being professor of mathematics. The teaching of mathematics in the military and naval schools is sometimes criticised for being much behind the times. The mathematics of the Nautical School of Tokyo is almost the same as in the naval schools.

Finally a few words may be said as to the publication of treatises and periodicals which are undoubtedly contributing largely to the improvement of the knowledge of mathematical studies. Although there is no work worthy of exceptional mention, yet the most interesting is certainly the series of treatises published by T. Hayashi. Books of advanced nature have begun to appear occasionally. We shall not however go into details with respect to Japanese manuals.

Among the mathematical journals published in Japan we may mention the *Journal of the Tokyo Physics School*, the *Mathematical Club*, the *Mathemathecal Journal*, the *Mathematical World*, the *XY*, etc. These are, with the exception of the first named, mostly read by the pupils and graduates of middle class schools. The proceedings of the Tokyo Mathematico-Physical Society contains original contributions, mostly composed in European languages. If one desires to know about the progress of mathematics recently made in Japan, he is requested to consult the mathematical journals. The contributors of mathematical essays may be mentioned: Endo, Fujisawa, Fujiwara, Fukuzawa, Hayashi, Kaba, Kaibara, Kąriya, Kato, Kikuchi, Kimura, Kitao, Kubota, Kumamoto, Mikami, Miwa, Miwada, Mizuhara, Motoda, Naito, Nakagawa, Ogura, Sudo, Sakai, Sawada, Sawayama, Takagi, Terao, Tsuruta, Yoshiye.

The essays of these authors concern various branches of mathematics, some being historical writings.

The Journal of the College of Science, Tokyo, contains also some articles relating to mathematies, the authors being Fujisawa, Nakagawa, Sakai, Sudo, and Takagi.

Besides, the essays contributed by Japanese mathematicians to foreign periodicals, though few in number, are also to be noted. The writings of Fukuzawa and others cannot escape our notice.

The academic degrees are conferred in Japan by the minister of education upon four classes of candidates: (1) those who have successfully

completed their studies in the Graduate Colleges of Imperial Universities, (2) those who have presented their theses, (3) those who have been recommended by the president of the Imperial University from among the professors. T. Takagi is the only mathematician who has ever received his doctor's degree in Japan through the presentation of a thesis. Among the holders of the same degree in the domain of mathematics we may count R. Fujisawa, J. Kawai, D. Kikuchi, the late D. Kitao (learned in mechanics), K. Miwa, and H. Terao (astronomer).

In Japan the publication of articles does not contribute to elevate the author's social position or improve his reputation, and there are many who have never published anything concerning their special studies and yet who have a reputation as learned men and consequently obtain their high positions. This custom will seem very odd to Americans and Europeans, who are full of the spirit of progress. This is certainly a partial reason why the progress of mathematics in Japan has not been as rapid as could be expected or hoped.